Ranking Treatments Using Instrumental Variables and Alternative Monotonicity Restrictions

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Introduction

Setup and Notation

Assumptions

Results

This paper studies treatment effects in the context of

- Discrete, possibly multi-valued treatments,
- Discrete, possibly multi-valued instrumental variables,
- Binary outcome variable.

Framework nests both ordered and unordered treatments.

The object of interest is either

- Pairwise average treatment effects (ATEs), or
- Rank ordering of treatments by average potential outcomes.

e.g., consider an RCT with (one-sided) noncompliance.

- Typical approaches analyze intention to treat (ITT) or LATE parameters.
- In some settings, these parameters may be less policy-relevant.
- Possible that treatment A > B for everyone, but ITT/LATE is smaller.

e.g., Angrist, Lang & Oreopoulos (2009):

Outcome = academic performance.

Treatment A = signing up to receive emails about tutoring.

Treatment B = signing up to receive emails about fellowship.

Instrument = offer to sign up.

Relevant policy is sending emails, not offers to sign up.

We apply method to four multi-arm RCTs:

Angrist, Lang & Oreopoulos (2009).

Behaghel, Crepon & Gurgand (2013, 2014).

  Effects of public vs. private job search assistance.

Blattman, Jamison & Sheridan (2017).

  Effects of cash incentives and therapy on reducing crime in Liberia.

Kline and Walters (2016).

  Effects of alternative early childhood programs.

Observed quantities: (Y, D, Z)

- $Y \in \{0, 1\}$: observed outcome.
- $D \in \mathcal{D} = \{0, \ldots, J-1\}$: treatment.
- $Z \in \mathcal{Z} = \{0, \ldots, J'-1\}$: instrument.

e.g., in RCT, could have $J = J'$ and

- Y: indicator variable for outcome.
- $D = j$: take-up of treatment j.
- $Z = j$: random assignment/encouragement to treatment j.
- Imperfect compliance leads to $D \neq Z$.

Latent variables:

Potential outcomes $\{Y_d\}_{d \in \mathcal{D}}$.

$$Y = \sum_{d \in \mathcal{D}} Y_d I\{D = d\} .$$

Potential treatments $\{D_z\}_{z \in \mathcal{Z}}$.

$$D = \sum_{z \in \mathcal{Z}} D_z I\{Z = z\} .$$

Let P denote distribution of (Y, D, Z).

Let Q denote distribution of $(\{Y_d\}_{d\in\mathcal{D}}, \{D_z\}_{z\in\mathcal{Z}}, Z)$.

$(Y, D, Z) = T(\{Y_d\}_{d\in\mathcal{D}}, \{D_z\}_{z\in\mathcal{Z}}, Z)$, so $P = QT^{-1}$.

Will restrict $Q \in \mathbf{Q}$ with alternative restrictions on $\mathbf{Q}$.

e.g., $E_Q[Y_j] \leq E_Q[Y_k]$ or $Y_j \leq Y_k$ with prob. 1.

Interested in ordering of $E_Q[Y_d], d \in \mathcal{D}$.

$\Pi =$ set of all permutations of $(0, \dots, J-1)$.

Say $E_Q[Y_d]$ follows $\pi = (\pi_0, \dots, \pi_{J-1})$, or $Q \in \mathbf{Q}^E_\pi$, if

$$E_Q[Y_{\pi_0}] \leq \dots \leq E_Q[Y_{\pi_{J-1}}] .$$

Under alternative restrictions $\mathbf{Q}$, we characterize

$\mathbf{Q}T^{-1}$, i.e., testable restrictions on P implied by $\mathbf{Q}$.

$\Delta_{jk}(P|\mathbf{Q}) = \{E_Q[Y_j - Y_k] : P = QT^{-1}, Q \in \mathbf{Q}\}$, i.e.,

…identified set of pairwise ATEs.

$\Pi(P) = \{\pi \in \Pi : P \in \mathbf{Q}_\pi^E T^{-1}\}$.

All results depend on analytical expressions (e.g., inequalities) involving P.

Study inference for model validity, ATEs, and ordering of treatments.

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We require the instrument Z to be exogenous:

Assumption 1 (Instrument Exogeneity)

$Z \perp\!\!\!\perp (\{Y_d\}_{d \in \mathcal{D}}, \{D_z\}_{z \in \mathcal{Z}})$ under Q.

In addition, we impose alternative restrictions on

- Treatment response types and
- Outcome response types.

Software being developed to allow practitioners to specify restrictions.

Treatment response types: how Z affects D.

Type is $r^t = (d_0, \dots, d_{J'-1}) \in \mathcal{D}^{J'}$.

Restrict $r^t \in R^t \subset \mathcal{D}^{J'}$.

Implies $Q \in \mathbf{Q}^{R^t}$, where

$$\mathbf{Q}^{R^t} = \{Q \in \mathbf{Q} : Q\{(D_0, \dots, D_{J'-1}) = r^t\} = 0, \forall r^t \notin R^t\} .$$

R^t implied by monotonicity restrictions depending on applied context.

Example: treatment response types for encouragement designs.

Consider a multi-arm RCT with noncompliance.

$J' = J$ so $\mathcal{D} = \mathcal{Z}$.

$Z = k$: random assignment to encouragement of treatment k.

Encouragement towards k weakly pushes subjects to treatment k.

Assumption 2 (Monotonicity of D in Z for Encouragement Design)

$$R^t = \{(d_0, \dots, d_{J-1}) : \exists j \in \mathcal{D} \text{ and } \Lambda \subset \mathcal{D} \backslash \{j\} \text{ s.t. } d_k = k \text{ for } k \in \Lambda \text{ and } d_k = j \text{ for } k \notin \Lambda\}\ .$$

When $J = 2$, $R^t = \{(0,0), (0,1), (1,1)\}$.

Equivalent to LATE monotonicity of Imbens and Angrist (1994).

Assumption 2 (Monotonicity of D in Z for Encouragement Design)

$$R^t = \{(d_0, \dots, d_{J-1}) : \exists j \in \mathcal{D} \text{ and } \Lambda \subset \mathcal{D}\backslash\{j\} \text{ s.t.}$$
$$d_k = k \text{ for } k \in \Lambda \text{ and } d_k = j \text{ for } k \notin \Lambda\}.$$

When $J = 3$,

$$R^t = \{(0,1,2), (0,1,0), (0,1,1), (0,0,2),$$
$$(0,2,2), (1,1,2), (2,1,2), (0,0,0), (1,1,1), (2,2,2)\}.$$

e.g., $(2,1,0)$, $(1,1,0)$ are not allowed.

$$R^t = \{(0,1,2),(0,1,0),(0,1,1),(0,0,2), \\ (0,2,2),(1,1,2),(2,1,2),(0,0,0),(1,1,1),(2,2,2)\}.$$

Restriction across triplets of potential choices.

Stronger than pairwise restrictions.

e.g., pairwise monotonicity implies $D_0 = 1 \Rightarrow D_1 = 1, \dots$

…while the above implies $D_0 = 1 \Rightarrow D_1 = 1$ and $D_2 \in \{1, 2\}$.

Nests RCT with one-sided non-compliance:

$$R^t = \{(d_0, \dots, d_{J-1}) : d_j \in \{0, j\}\}.$$

Of course could consider other R^t.

Outcome response types: how D affects Y

Type is $r^o = (y_0, \dots, y_{J-1}) \in \{0,1\}^J$.

Restrict $r^o \in R^o \subset \{0,1\}^J$.

Implies $Q \in \mathbf{Q}^{R^o}$, where

$$\mathbf{Q}^{R^o} = \{Q \in \mathbf{Q} : Q\{(Y_0, \dots, Y_{J-1}) = r^o\} = 0, \forall r^o \notin R^o\} .$$

Simultaneously restrict treatment/outcome response types:

$$\mathbf{Q}^{R^t, R^o} = \mathbf{Q}^{R^t} \cap \mathbf{Q}^{R^o} .$$

Example: outcome response types in Angrist, Lang & Oreopoulos (2009).

Multi-arm RCT with random assignment to

$D = 0$: untreated.

$D = 1$: treated with tutoring.

$D = 2$: treated with fellowship.

$D = 3$: treated with both tutoring and fellowship.

Y is measure of academic success.

Natural to impose $Y_3 \geq Y_j \geq Y_0$, $j = 1, 2$.

i.e., neither tutoring nor fellowship can hurt.

So $R^o = \{(y_0, y_1, y_2, y_3) : y_3 \geq y_j \geq y_0, j = 1, 2\}$.

Theorem 1

Under Assumption 1, for any $R^t \subset \mathcal{D}^{\mathcal{Z}}$ and $R^o \subset \{0,1\}^J$,

(i) $\mathbf{Q}^{R^t,R^o} T^{-1} = \mathbf{P}^{R^t,R^o}$, where $\mathbf{P}^{R^t,R^o}$ is bounded polyhedron defined by finite number of linear inequalities.

(ii) For $j \neq k \in \mathcal{D}$, $\Delta_{jk}(P|\mathbf{Q}^{R^t,R^o})$ is either empty set or closed interval.

Theorem 1(i) is about testable restrictions.

Theorem 2(ii) is about identified sets of pairwise ATEs.

Inequalities and bounds depend on analytical expressions involving

$$p_{ydz} = P\{Y = y, D = d | Z = z\}\,.$$

Could easily incorporate $Q \in \mathbf{Q}_{\pi}^{E}$ as additional constraints.

Theorem 2

If the feasible sets under (R^t, R^o) and $(\tilde{R}^t, R^o)$ are both nonempty, then $\Delta_{jk}(P|\mathbf{Q}^{R^t,R^o}) = \Delta_{jk}(P|\mathbf{Q}^{\tilde{R}^t,R^o})$.

i.e., for two treatment response types, either

- Identified sets of ATEs are the same, or
- One of them is empty.

Restricting treatment response types does not shorten identified sets!

Now study encouragement design.

Corollary 3

Under Assumptions 1–2,

(i) $\mathbf{Q}^{R^t} T^{-1} = \mathbf{P}^{R^t}$, bounded polyhedron defined by 64 linear inequalities.

(ii) For $j \neq k \in \mathcal{D}$,

$$\Delta_{jk}(P|\mathbf{Q}^{R^t}) = \begin{cases} \emptyset & \text{if } P \notin \mathbf{P}^{R^t} \\ [p_{0kk} + p_{1jj} - 1, 1 - p_{0jj} - p_{1kk}] & \text{if } P \in \mathbf{P}^{R^t} \end{cases},$$

where $p_{ydz} = P\{Y = y, D = d | Z = z\}$.

(iii) For $\pi \in \Pi$ and $\mathbf{P}^E_\pi = \mathbf{Q}^E_\pi T^{-1}$,

$$\mathbf{P}^{R^t} \cap \mathbf{P}^E_\pi = \mathbf{P}^{R^t} \cap \{P \in \mathbf{P} : p_{0\pi_k\pi_k} + p_{1\pi_j\pi_j} - 1 \leq 0 \text{ for } j < k \in \mathcal{D}\}.$$

Corollary 3(i): Testable restriction $P \in \mathbf{P}^{R^t}$ substantially more restrictive

…than Assumption 1 + pairwise monotonicity.

Pairwise monotonicity $D_k = j \Rightarrow D_j = j$ implies for any $y \in \{0, 1\}$,

$$\{Y_j = y, D_k = j\} \subset \{Y_j = y, D_j = j\} \Rightarrow$$
$$P\{Y_j = y, D_k = j\} \leq P\{Y_j = y, D_j = j\} \Rightarrow$$
$$P\{Y = y, D = j | Z = k\} \leq P\{Y = y, D = j | Z = j\}\,.$$

There are 12 inequalities of that kind.

Looking at triplets generates 52 more nonredundant inequalities.

Corollary 3(ii):

$$\Delta_{jk}(P|\mathbf{Q}^{R^t}) = \begin{cases} \emptyset & \text{if } P \notin \mathbf{P}^{R^t} \\ [p_{0kk} + p_{1jj} - 1, 1 - p_{0jj} - p_{1kk}] & \text{if } P \in \mathbf{P}^{R^t} \end{cases} .$$

If impose Assumption 1 alone, no restriction on R^t,

Then, bound either empty or same as above, shown in Manski (1990).

Corollary of Theorem 2.

Analogous to results for $J' = J = 2$ from Balke and Pearl (1995, 1997).

Restricing outcome response types.

Recall Angrist, Lang & Oreopoulos (2009).

$D = 0$: untreated.

$D = 1$: treated with tutoring.

$D = 2$: treated with fellowship.

$D = 3$: treated with both tutoring and fellowship.

Assume neither tutoring nor fellowship can hurt.

$R^o = \{(y_0, y_1, y_2, y_3) : y_3 \geq y_j \geq y_0, j = 1, 2\}$.

$R^t = \{(d_0, \ldots, d_{J=1}) : d_j \in \{0, j\}\}$, one-sided noncompliance.

Corollary 4

Under Assumption 1, suppose $J = 4$. For R^t, R^o given above,

(i) $\mathbf{Q}^{R^t,R^o} T^{-1} = \mathbf{P}^{R^t,R^o}$, where $\mathbf{P}^{R^t,R^o}$ is a bounded polyhedron defined by a finite number of linear inequalities.

(ii)

$$\Delta_{21}(P|\mathbf{P}^{R^t,R^o}) = \begin{cases} \emptyset & \text{if } P \notin \mathbf{P}^{R^t,R^o} \\ [\max\{p_{033}, p_{011}\} - p_{002} - p_{022}, & \\ p_{001} + p_{011} - \max\{p_{033}, p_{022}\}] & \text{if } P \in \mathbf{P}^{R^t,R^o} \end{cases},$$

with the identified sets for the remaining ATEs given in the paper.

Width = $p_{001} + p_{002} - \max\{p_{033} - p_{022}, 0\} - \max\{p_{033} - p_{011}, 0\}$.

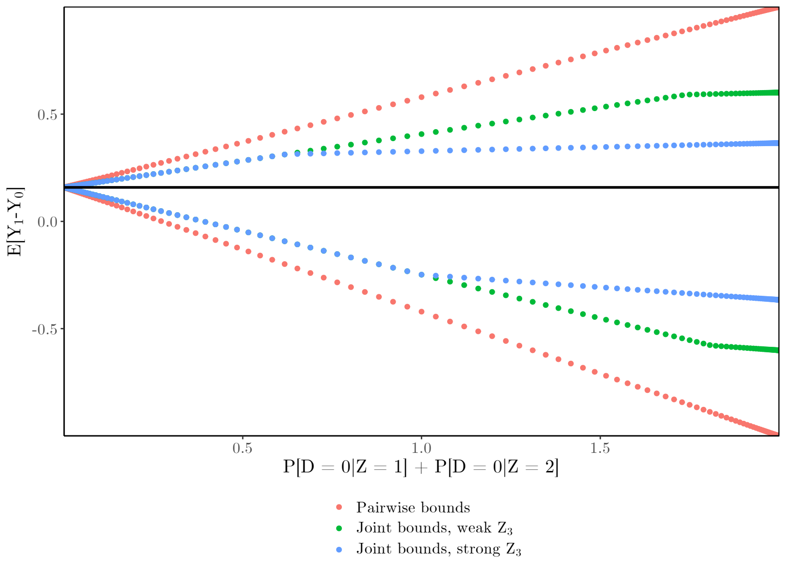
Width without restricting R^y is $P\{D = 0|Z = 1\} + P\{D = 0|Z = 2\}$.

Difference = $p_{102} + p_{101} + \max\{p_{033} - p_{022}, 0\} + \max\{p_{033} - p_{011}, 0\}$.

- Difference positive, though restriction is not directly on Y_1 vs Y_2.
- Increasing in noncompliance of 1 and 2, $P\{D = 0|Z = j\}, j = 1, 2$.
- Increasing in strength of instrument 3, $P\{D = 3|Z = 3\}$.

Next report results for numerical illustration.

RESULTS (CONT.)



Angrist et al. (2009), female subsample, for $E[Y_2 - Y_1]$.

- Imposing only one-sided non-compliance:
  - Width of sample analog to identified set is 0.51.
  - The estimated identified set is $[-0.18, 0.33]$.
  - 95% confidence set for $E[Y_2 - Y_1]$ is $[-0.27, 0.46]$.
- Additionally imposing restriction on outcome response types:
  - Width of sample analog to identified set is 0.02.
  - The estimated identified set is $[-0.02, 0.00]$.
  - 95% confidence set for $E[Y_2 - Y_1]$ is $[-0.14, 0.11]$.
- Identified sets and CIs much shorter under restrictions on R^y.

Framework applies to a wide class of examples.

Assume discrete with small support for convenience.

LP structure + recent advances on inference $\Rightarrow$ handle richer examples.

Appendix

Stronger than "unordered monotonicity" of Heckman and Pinto (2018):

for all $d \in \mathcal{D}$ and all $z, z' \in \mathcal{Z}$, either $I\{D_z = d\}$

$\geq I\{D_{z'} = d\}$w.p.1 or $I\{D_{z'} = d\} \geq I\{D_z = d\}$w.p.1 .

e.g., $R^t = \{(0,1,1),(2,1,1)\}$ doesn't violate their assumption.

Distinct from mono. condition of Behaghel, Crepon & Gurgand (2013):

$$D_0 = 1 \Leftrightarrow D_2 = 1$$

$$D_0 = 2 \Leftrightarrow D_1 = 2$$

$$D_1 = 0 \Leftrightarrow D_2 = 0 \, .$$

RCT for binary treatment with a close substitute: Kline and Walters (2016).

Randomization of vouchers for treatments: Heckman and Pinto (2018)

Voucher possibly good for more than one treatment, with WARP used to restrict response types.

Ordered treatment: Angrist and Imbens (1995), Heckman, Vytlacil & Urzua (2006).

EXAMPLE OF INEQUALITY IMPLIED BY TRIPLETS BUT NOT PAIRS

$$
\begin{aligned}
&(P\{D=0|Z=1\}-P\{D=0|Z=0\}) \\
&\qquad +(P\{Y=0,D=1|Z=2\}-P\{Y=0,D=1|Z=0\}) \\
&=(P\{D_1=0\}-P\{D_0=0\})+(P\{Y_1=0,D_2=1\}-P\{Y_1=0,D_0=1\})
\end{aligned}
$$

$$
\begin{aligned}
&(P\{D=0|Z=1\} - P\{D=0|Z=0\}) \\
&\qquad + (P\{Y=0, D=1|Z=2\} - P\{Y=0, D=1|Z=0\}) \\
&= (P\{D_1=0\} - P\{D_0=0\}) + (P\{Y_1=0, D_2=1\} - P\{Y_1=0, D_0=1\})
\end{aligned}
$$

Using instrument exogeneity.

EXAMPLE OF INEQUALITY IMPLIED BY TRIPLETS BUT NOT PAIRS

$$
\begin{aligned}
&(P\{D = 0|Z = 1\} - P\{D = 0|Z = 0\}) \\
&\qquad + (P\{Y = 0, D = 1|Z = 2\} - P\{Y = 0, D = 1|Z = 0\}) \\
&= (P\{D_1 = 0\} - P\{D_0 = 0\}) + (P\{Y_1 = 0, D_2 = 1\} - P\{Y_1 = 0, D_0 = 1\})
\end{aligned}
$$

Using instrument exogeneity.

EXAMPLE OF INEQUALITY IMPLIED BY TRIPLETS BUT NOT PAIRS

$$
\begin{aligned}
&(P\{D=0|Z=1\}-P\{D=0|Z=0\}) \\
&\qquad +(P\{Y=0,D=1|Z=2\}-P\{Y=0,D=1|Z=0\}) \\
&=(P\{D_1=0\}-P\{D_0=0\})+(P\{Y_1=0,D_2=1\}-P\{Y_1=0,D_0=1\}) \\
&=(P\{D_1=0,D_0=0\}-P\{D_0=0,D_1=1\}-P\{D_0=0,D_1=0\})
\end{aligned}
$$

$D_1=0 \Rightarrow D_0=1.$

$$\begin{aligned}
&(P\{D=0|Z=1\}-P\{D=0|Z=0\}) \\
&\qquad +(P\{Y=0,D=1|Z=2\}-P\{Y=0,D=1|Z=0\}) \\
&=(P\{D_1=0\}-P\{D_0=0\})+(P\{Y_1=0,D_2=1\}-P\{Y_1=0,D_0=1\}) \\
&=(\cancel{P\{D_1=0,D_0=0\}}-P\{D_0=0,D_1=1\}-\cancel{P\{D_0=0,D_1=0\}})
\end{aligned}$$

Cancelling terms.

EXAMPLE OF INEQUALITY IMPLIED BY TRIPLETS BUT NOT PAIRS

$$
\begin{aligned}
&(P\{D = 0|Z = 1\} - P\{D = 0|Z = 0\}) \\
&\qquad + (P\{Y = 0, D = 1|Z = 2\} - P\{Y = 0, D = 1|Z = 0\}) \\
&= (P\{D_1 = 0\} - P\{D_0 = 0\}) + (P\{Y_1 = 0, D_2 = 1\} - P\{Y_1 = 0, D_0 = 1\}) \\
&= -P\{D_0 = 0, D_1 = 1\} \\
&\qquad + P\{Y_1 = 0, D_2 = 1, D_1 = 1, D_0 = 1\} + P\{Y_1 = 0, D_2 = 1, D_1 = 1, D_0 = 0\} \\
&\qquad - P\{Y_1 = 0, D_0 = 1, D_1 = 1, D_2 = 1\} - P\{Y_1 = 0, D_0 = 1, D_1 = 1, D_2 = 2\}
\end{aligned}
$$

$D_2 = 1 \Rightarrow D_1 = 1, D_0 \in \{0, 1\}$. $D_0 = 1 \Rightarrow D_1 = 1, D_2 \in \{1, 2\}$.

$$\begin{aligned}
&(P\{D = 0|Z = 1\} - P\{D = 0|Z = 0\}) \\
&\qquad + (P\{Y = 0, D = 1|Z = 2\} - P\{Y = 0, D = 1|Z = 0\}) \\
&= (P\{D_1 = 0\} - P\{D_0 = 0\}) + (P\{Y_1 = 0, D_2 = 1\} - P\{Y_1 = 0, D_0 = 1\}) \\
&= -P\{D_0 = 0, D_1 = 1\} \\
&\qquad + \cancel{P\{Y_1 = 0, D_2 = 1, D_1 = 1, D_0 = 1\}} + P\{Y_1 = 0, D_2 = 1, D_1 = 1, D_0 = 0\} \\
&\qquad - \cancel{P\{Y_1 = 0, D_0 = 1, D_1 = 1, D_2 = 1\}} - P\{Y_1 = 0, D_0 = 1, D_1 = 1, D_2 = 2\}
\end{aligned}$$

Cancelling terms.

$$
\begin{aligned}
&(P\{D=0|Z=1\}-P\{D=0|Z=0\})\\
&\qquad +(P\{Y=0,D=1|Z=2\}-P\{Y=0,D=1|Z=0\})\\
&=(P\{D_1=0\}-P\{D_0=0\})+(P\{Y_1=0,D_2=1\}-P\{Y_1=0,D_0=1\})\\
&=-P\{D_0=0,D_1=1\}+P\{Y_1=0,D_2=1,D_1=1,D_0=0\}\\
&\qquad -P\{Y_1=0,D_0=1,D_1=1,D_2=2\}\\
&\leq 0\,.
\end{aligned}
$$

$\{Y_1=0,D_2=1,D_1=1,D_0=0\}\subset\{D_1=1,D_0=0\}$.

$$
\begin{aligned}
&(P\{D=0|Z=1\}-P\{D=0|Z=0\}) \\
&\qquad\qquad +(P\{Y=0,D=1|Z=2\}-P\{Y=0,D=1|Z=0\}) \\
&=(P\{D_1=0\}-P\{D_0=0\})+(P\{Y_1=0,D_2=1\}-P\{Y_1=0,D_0=1\}) \\
&=-P\{D_0=0,D_1=1\}
\end{aligned}
$$

$\Rightarrow p_{001}+p_{101}-p_{000}-p_{100}+p_{012}-p_{010}\leq 0.$